Calculation Policy

Reviewed: Autumn 2017
Review Date: Spring 2019

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Headteacher	Chair of Governors	Date

Mastery in Maths

The Mastery-learning model forms the basis of our approach to Mathematics teaching. This means spending greater time going into depth about a subject as opposed to racing through the things that all children should know. Previously, racing through content lead to some children having large gaps in subject knowledge because the concept they had just learnt was either too big or learnt too quickly. As a primary school, it is our duty to ensure that children have an absolutely solid, concrete understanding of subject knowledge and skills as well as being emotionally resilient for secondary school. With calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.

Now, we have the confidence to take learning at a steadier and deeper pace, ensuring that no child is left behind, as well as providing deeper and richer experiences for children who are above the national expectation for their age.

We focus on all children achieving what is expected of their age group and not going beyond this. Evidence shows that children need to be able to understand a concept, apply it in a range of situations and then be creative to really understand it. Simply going beyond their age group does not guarantee they understand something, it just means they have heard it.

At our school no child will be taught content from the year group above them, they will spend time becoming true masters of content, applying and being creative with new knowledge and skills in multiple ways.

In short, this means working towards:
-Teach less, learn more: less teacher talk and more evidencing learning and progress
-No child left behind: all children are enabled to keep up every day.
-Space and time to experience and apply, with all children entitled to additional support to ensure they do not fall behind or to go deeper
-Understanding real life applications wherever possible to make learning relevant and not abstract; nothing should be taught without a purpose.

All of this means that you may see a change in the way we teach and assess your child, most notably will be in how we organise your child's learning and how we report their progress to you.

We will be doing more of this:
-Teaching all children in class, together, most of the time
-Verbal feedback during lessons, shorted comments in books and more ticking of correct concepts
-Spending longer on one idea
-Giving children who need it, additional support over shorter, more intense periods, like a day or week.

This policy outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Progression in Calculation

Introduction

At Featherstone Wood Primary & Nursery School we believe that children should be introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave Year 6 is that they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to 20 (Year 2), sums and differences of multiples of 4, 8 50 and 100 (Year 3) and multiplication facts up to 12 × 12 (Year 4);

Written methods of calculation

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that we adopt greater consistency in our approach to calculation. The challenge is for our teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

Objectives

The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'.

Using and applying mathematics	**Calculating**
EYFS . Number knowledge based on the Early Learning Goals Outcomes.	**EYFS** . Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. . Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. . They solve problems, including doubling, halving and sharing.
Year 1 • Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change' • Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	**Year 1** • Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number • Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number • Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences
Year 2 • Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence • Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem	**Year 2** • Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders • Use the symbols +, –, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. □ ÷ 2 = 6, 30 – □ = 24)

Year 3 • Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations • Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or ones of measure	**Year 3** • Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers • Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context • Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences
Year 4 • Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate • Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem	**Year 4** • Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p • Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6)
Year 5 • Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use • Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem	**Year 5** • Use efficient written methods to add and subtract whole numbers and decimals with up to two places • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000 • Refine and use efficient written methods to multiply and divide THHTO × O, TO × TO, O.t × O and THHTO ÷ O
Year 6 • Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use • Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of c pens at 15 pence each is $15c$ pence)	**Year 6** • Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

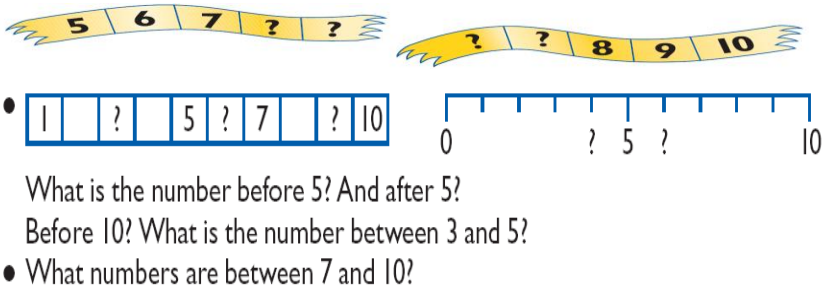
Written methods for addition of whole numbers

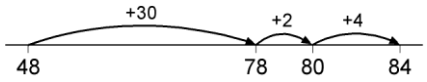
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 3.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

EYFS Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.	 Various manipulatives are used to develop counting and number recognition. For example, digit cards, Numicon, which leads to using number tracks.

Phase 1: The empty number line • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	**Phase 1** Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15  48 + 36 = 84  or:  **This will also include the use of the 100 square to reinforce the use of partitioning.**
Phase 2: Partitioning • The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.	**Phase 2** Record steps in addition using partitioning: 47 + 76 = 47 + 70 + 6 = 117 + 6 = 123 47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123 Partitioned numbers are then written under one another: 47 40 + 7 76 + = 70 + 6 110+13 = 123 **Teaching point** Ensure correct use of the = sign.
Phase 3: Expanded method in columns • Move on to a layout showing the addition of the ones to the ones. To find the partial sums the ones are added first and the total of the partial sums can be found by adding them in any order. • The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	**Phase 3** Write the numbers in columns. Adding the ones first: 47 76 + 13 110 + 123

Phase 4: Column method • In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	**Phase 4** $\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \scriptstyle 1\,1 \end{array}$ $\begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \scriptstyle 1\,1 \end{array}$ $\begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \scriptstyle 1\,1 \end{array}$ Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.
Phase 5: Column method expanded to decimals In this phase the standard method is expanded to include the use of decimals.	

Written methods for subtraction of whole numbers

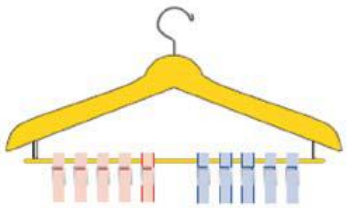
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

These notes show the Phase s in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 3.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact, 16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

EYFS Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.	    1 less than 10 is 9 10 substract 1 equals 9 10 - 1 = 9

Phase 1: Using the empty number line

- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

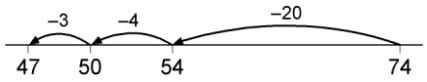
The notes below give more detail on the counting-up method using an empty number line.

Phase 1

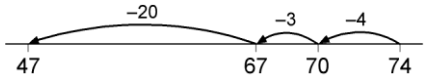
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 – 7 = 8

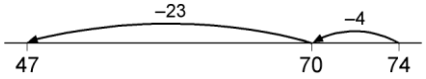
74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:



or combined:



Phase 2: Partitioning

- Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For
  74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.

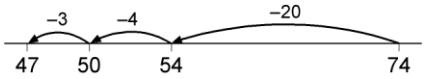
Phase 2

Subtraction can be recorded using partitioning:

74 – 27 = 74 – 20 – 7 = 54 – 7 = 47

74 – 27 = 70 + 4 – 20 – 7 = 60 + 14 – 20 – 7 = 40 + 7

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.



Stage 3: Expanded layout, leading to column method

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Phase 3

Partitioned numbers are then written under one another:

Example: 74 − 27

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 60}\ {\scriptstyle 14} \\ \cancel{70}+\cancel{4} \\ -\ 20+7 \\ \hline 40+7 \end{array} \qquad \begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 14} \\ \cancel{7}\ \cancel{4} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

Example: 741 − 367

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 600}\ {\scriptstyle 130}\ {\scriptstyle 11} \\ \cancel{700}+\cancel{40}+\cancel{1} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array} \qquad \begin{array}{r} {\scriptstyle 6}\ {\scriptstyle 13}\ {\scriptstyle 11} \\ \cancel{7}\ \cancel{4}\ \cancel{1} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

The expanded method for three-digit numbers

Example: 563 − 241, no adjustment or decomposition needed

Expanded method leading to

$$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array} \qquad \begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \qquad \begin{array}{r} 400+160+3 \\ -\ 200+\ 70+1 \\ \hline 200+\ 90+2 \end{array} \qquad \begin{array}{r} {\scriptstyle 400}\ \ {\scriptstyle 160} \\ \cancel{500}+\cancel{60}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array} \qquad \begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 16} \\ \cancel{5}\ \cancel{6}\ 3 \\ -\ 2\ 7\ 1 \\ \hline 2\ 9\ 2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+150+13 \\ -\ 200+\ 70+\ 8 \\ \hline 200+\ 80+\ 5 \end{array} \qquad \begin{array}{r} {\scriptstyle 400}\ \ {\scriptstyle 150}\ \ {\scriptstyle 13} \\ \cancel{500}+\cancel{60}+\cancel{3} \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \qquad \begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 15}\ {\scriptstyle 13} \\ \cancel{5}\ \cancel{6}\ \cancel{3} \\ -\ 2\ 7\ 8 \\ \hline 2\ 8\ 5 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

$$\begin{array}{r} 500+\ 0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+\ 8 \\ \hline 200+20+\ 5 \end{array} \qquad \begin{array}{r} {\scriptstyle 400}\ \ {\scriptstyle 90}\ \ {\scriptstyle 13} \\ \cancel{500}+\cancel{0}+\cancel{3} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \qquad \begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 9}\ {\scriptstyle 13} \\ \cancel{5}\ \cancel{0}\ \cancel{3} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Teaching point;

Ensure that the pupils are secure at the expanded stage before progressing to the standard decomposition method

Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 3, two-digit by two-digit multiplication by the end of Year 4, and three-digit by two-digit multiplication by the end of Year 5.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

EYFS/KS1

Solving problems by doubling.

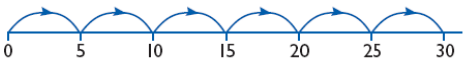
$2 + 2 + 2 + 2 + 2 = 10$

$2 \times 5 = 10$

2 multiplied by 5

5 pairs

5 hops of 2



$5 + 5 + 5 + 5 + 5 + 5 = 30$

$5 \times 6 = 30$

5 multiplied by 6

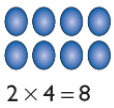
6 groups of 5

6 hops of 5

$10p + 10p + 10p + 10p + 10p = 50p$

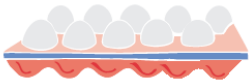
$10p \times 5 = 50p$

5 hops of 10



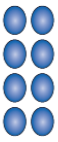
$4 \times 2 = 8$

$2 \times 4 = 8$



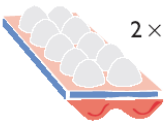
$5 \times 2 = 10$

$2 \times 5 = 10$



$2 \times 4 = 8$

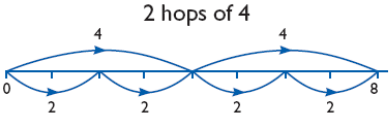
$4 \times 2 = 8$



$2 \times 5 = 10$

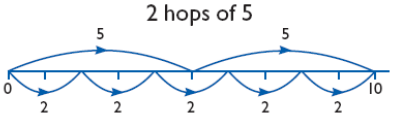
$5 \times 2 = 10$

2 hops of 4



4 hops of 2

2 hops of 5



5 hops of 2

Phase 1: Mental multiplication using partitioning • Mental methods for multiplying TO × O can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.	**Phase 1** Informal recording in Year 4 might be: 43 40 + 3 ↓ ↓ × 6 240 + 18 = 258 Also record mental multiplication using partitioning: $14 \times 3 = (10 + 4) \times 3$ $= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6$ $= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$ Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7: ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○
Phase 2: The grid method • As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps. • It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.	**Phase 2** 38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266 × | 7 30 | 210 8 | 56 | 266
Phase 3: Expanded short multiplication • The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above. • Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed. • Most children should be able to use this expanded method for TO × O by the end of Year 4.	**Phase 3** 30 + 8 X 7 56 8 x 7 210 + 30 x 7 266 38 X 7 56 210 + 266

Phase 4: Short multiplication • The recording is reduced further, with carry digits recorded below the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.	**Phase 4** 38 × 7 266 5 The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.
Phase 5: Two-digit by two-digit products • Extend to TO × TO, asking children to estimate first. • Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product. • As in the grid method for TO × O in stage 4, the first column can become an extra top row as a stepping stone to the method below.	**Phase 5** 56 × 27 is approximately 60 × 30 = 1800. × | 20 | 7 | 50 | 1000 | 350 | 1350 6 | 120 | 42 | 162 1512 1 50 | 6 × | 20 | 7 1000 | 350 | 1350 120 | 42 | 162 1512 1
• Reduce the recording, showing the links to the grid method above.	56 × 27 is approximately 60 × 30 = 1800. 56 X 27 42 6 x 7 120 6 x 20 350 50 x 7 1000 + 50 x 20 1512 1

• Reduce the recording further. • The carry digits in the partial products of 56 × 7 = 392 and 56 x 20 = 1120 are usually carried mentally. • The aim is for most children to use this long multiplication method for TO × TO by the end of Year 4.	56 × 27 is approximately 60 × 30 = 1800. 56 X 27 392 56 x 7 1120 + 56 x 20 1512 1
Phase 6: Three-digit by two-digit products • Extend to HTO × TO asking children to estimate first. Start with the grid method. • It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.	**Phase 6** 286 × 29 is approximately 300 × 30 = 9000. × | 20 | 9 | 200 | 4000 | 1800 | 5800 80 | 1600 | 720 | 2320 6 | 120 | 54 | 174 8294 1
• Reduce the recording, showing the links to the grid method above. • This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.	286 X 29 54 6 x 9 120 6 x 20 720 80 x 9 1600 80 x 20 1800 200 x 9 4000 + 200 x 20 8294 1
• Children who are already secure with multiplication for TO × O and TO × TO should have little difficulty in using the same method for HTO × TO. • Again, the carry digits in the partial products are usually carried mentally but can be positioned in the correct column.	286 × 29 is approximately 300 × 30 = 9000. 286 X 29 2574 5720 + 8294 1

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to long division through Years 3 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

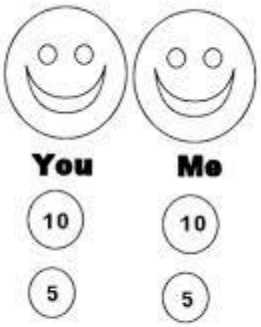
To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Solving problem by halving and sharing. **Children learn halving and sharing using pictorial problem solving techniques.**		

Phase 1: Mental division using partitioning

- Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.
- Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

Phase 1

One way to work out TO ÷ O mentally is to partition TO into a multiple of the divisor plus the remaining ones, then divide each part separately.

Informal recording in Year 4 for 84 ÷ 7 might be:

84

70 + 14

↓ ↓ ÷ 7

10 + 2 = 12

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Phase 2: 'Expanded' method for HTO ÷ O

- This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
- For TO ÷ O there is a link to the mental method.
- As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?'
- Once they understand and can apply the method, children should be able to move on from TO ÷ O to HTO ÷ O quite quickly as the principles are the same.
- This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
- Chunking is useful for reminding children of the link between division and repeated subtraction.
- However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

Phase 2

97 ÷ 9

$9\overline{)97}$

$-\ 90$ $\quad 9 \times 10$

7

Answer: $\quad 10\ \mathrm{R}\ 7$

$6\overline{)196}$

$-\ 60$ $\quad 6 \times 10$

136

$-\ 60$ $\quad 6 \times 10$

76

$-\ 60$ $\quad 6 \times 10$

16

$-\ 12$ $\quad 6 \times 2$

$4 \quad\quad 32$

Answer: $\quad 32\ \mathrm{R}\ 4$

- The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTO ÷ O involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTO dividend.
- Estimating has two purposes when doing a division:
  - to help to choose a starting point for the division;
  - to check the answer after the calculation.
- Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40.

Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.

$$
\begin{array}{rll}
6\overline{)196} & & \\
-\ 180 & 6\times 30 & \\
16 & & \\
-\ 12 & 6\times 2 & \\
4 & 32 & \\
\text{Answer:} & 32\,\text{R}\,4 &
\end{array}
$$

The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

Phase 3: Long division

The next step is to tackle HTO ÷ TO, which for most children will be in Year 5.

The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

Phase 3

Step 1

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$
\begin{array}{rll}
24\overline{)\ 560} & & \\
20\ -\ 480 & 24\times 20 & \\
80 & & \\
3\quad 72 & 24\times 3 & \\
8 & &
\end{array}
$$

Answer: 23 R 8

Step 2

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$
\begin{array}{r}
23 \\
24\overline{)\ 560} \\
-480 \\
80 \\
-72 \\
8
\end{array}
$$

Answer: 23 R 8

Step 3

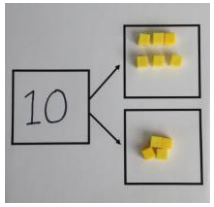
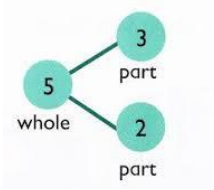
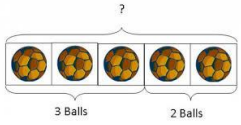
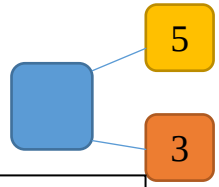
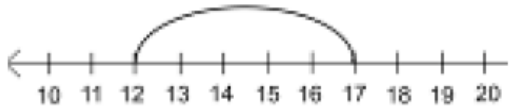
0 2 3 r8

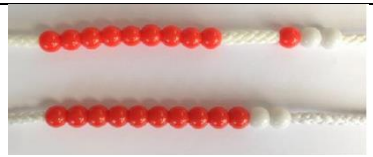
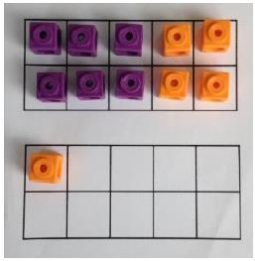
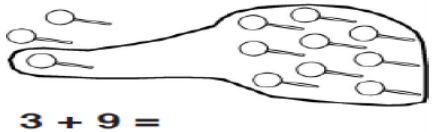
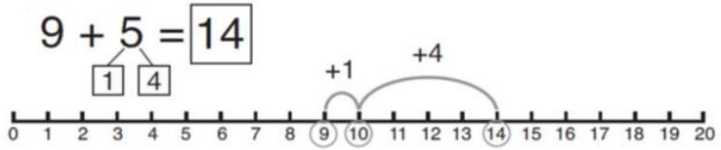
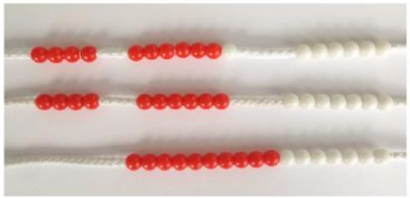
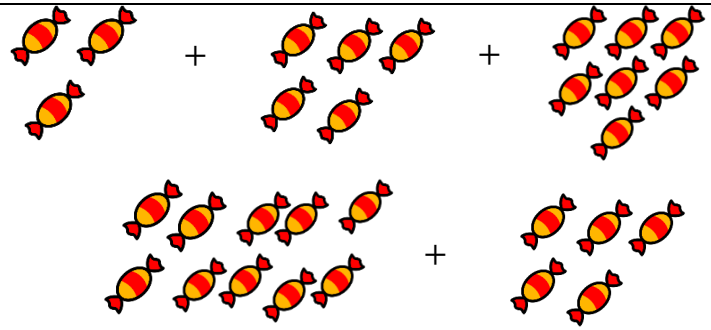
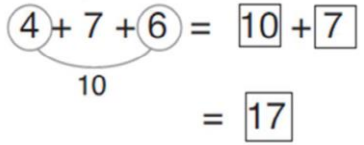
24)5 6 0

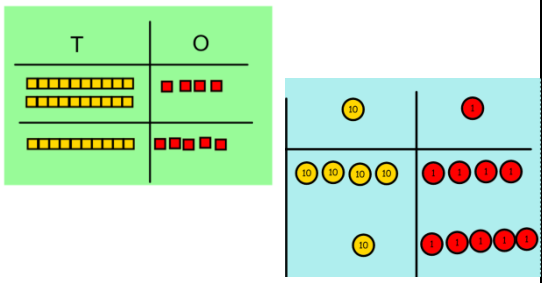
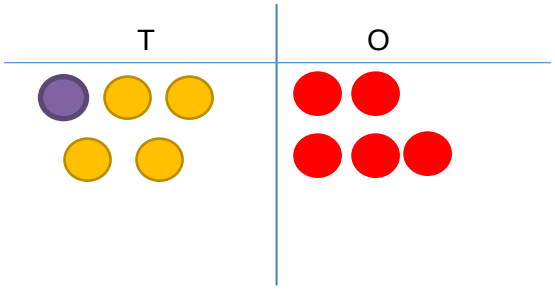
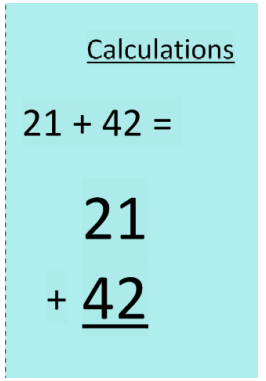
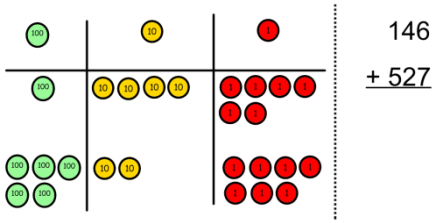
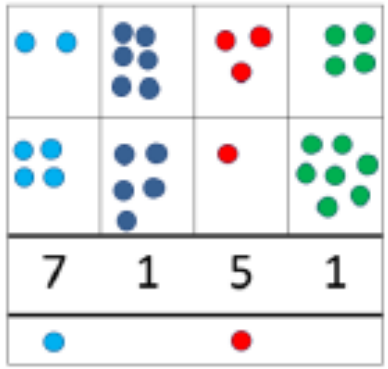
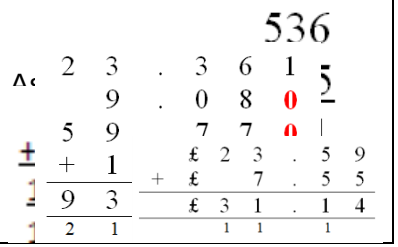
Appendix 1

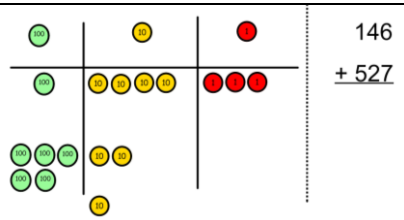
Progression in Calculations

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count	5 + 12 = 17 Place the larger number in your head and count on the

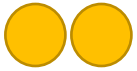
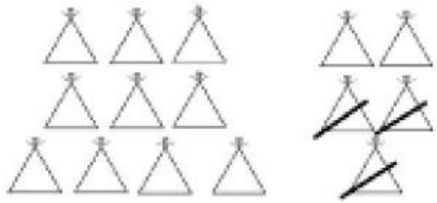
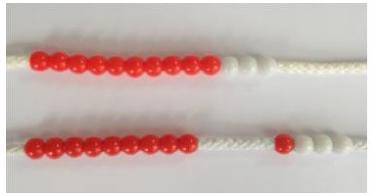
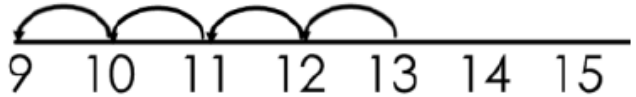
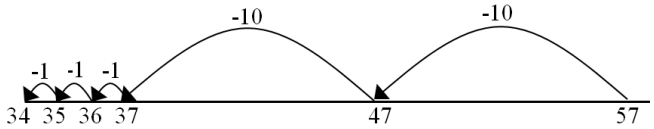
		on in ones or in one jump to find the answer.	smaller number to find your answer.
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.

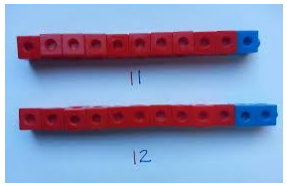
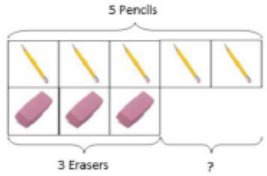
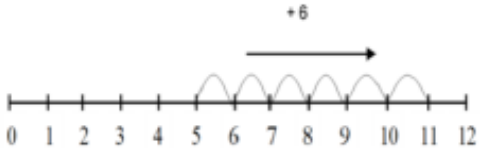
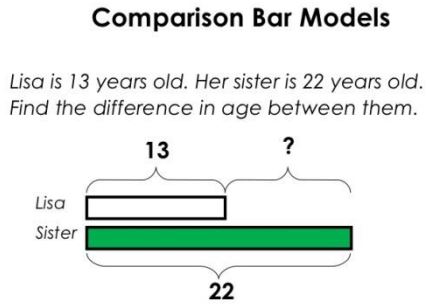
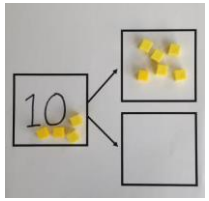
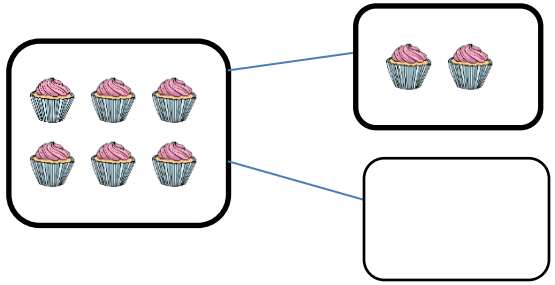
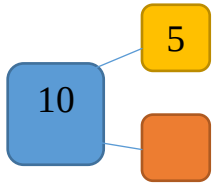
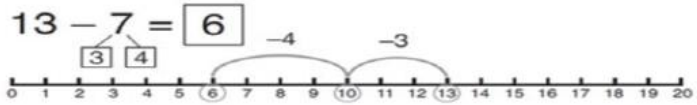
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	
Column method- regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. 	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 

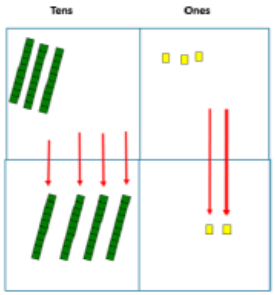
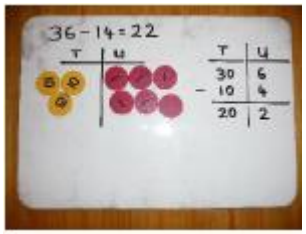
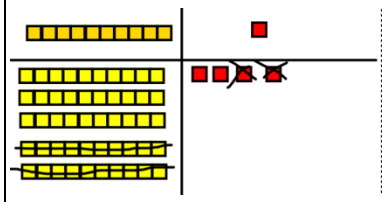
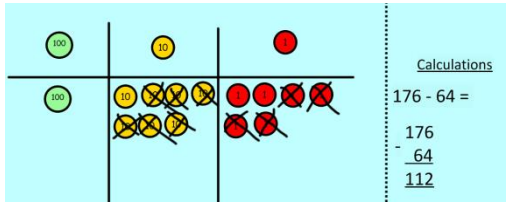
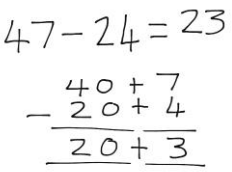
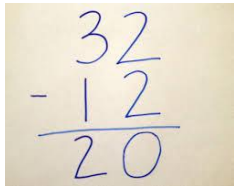
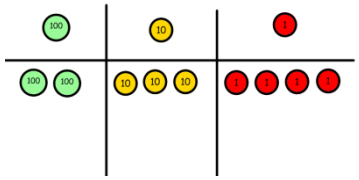
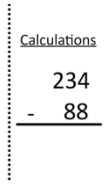
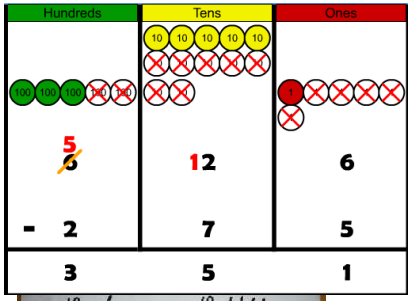
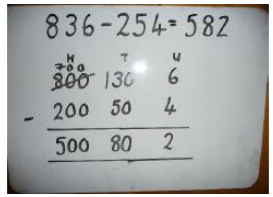
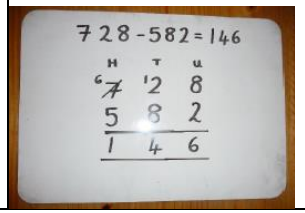
	 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.		the same number of decimal places and different. Money can be used here.

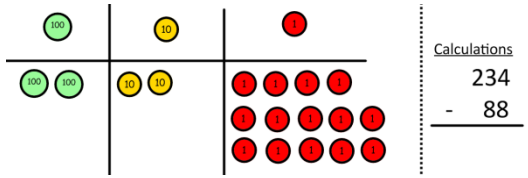
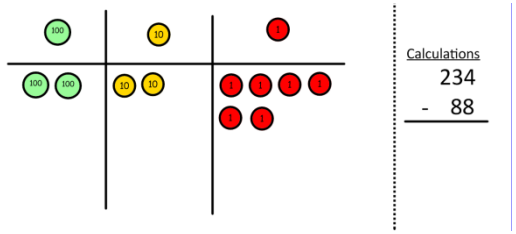
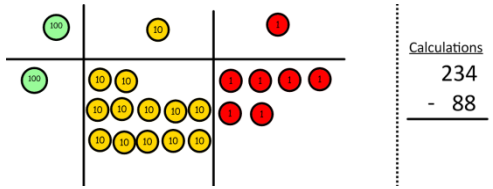
Subtraction

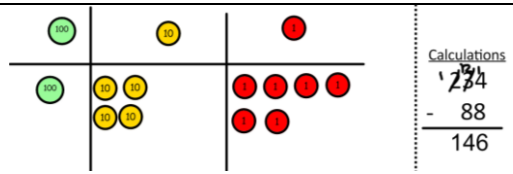
Objective and Strategies	Concrete	Pictorial	Abstract

Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

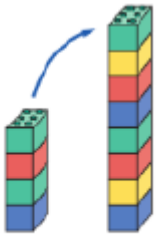
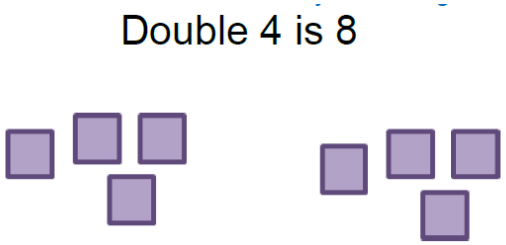
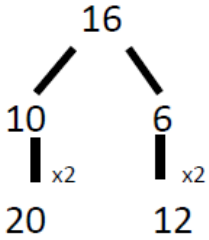
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.
Make 10	14 – 9 =  Make 14 on the ten frame. Take away	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You	16 – 8= How many do we take off to reach the next 10? How many do we have left

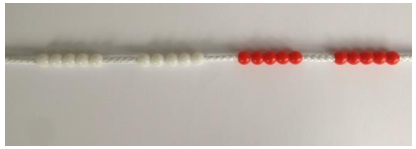
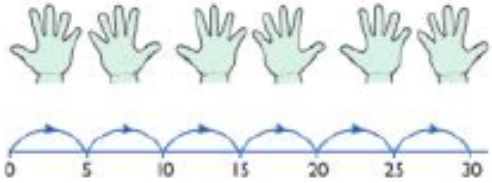
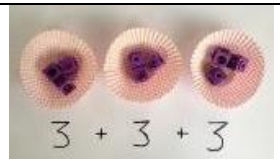
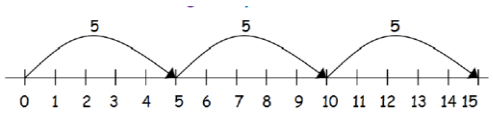
	the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	have reached your answer.	to take off?
Column method without regrouping	 Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the Base 10 or place value counters alongside the written calculation to help to show working. 	 This will lead to a clear written column subtraction. 
Column method with regrouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters   Start with the ones, can I take away 8	 Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping.	 Children can start their  formal written method by

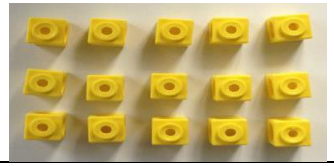
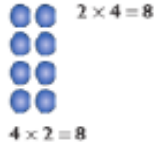
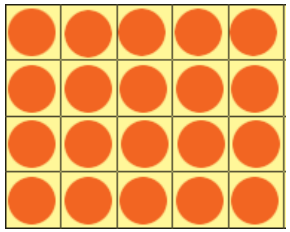
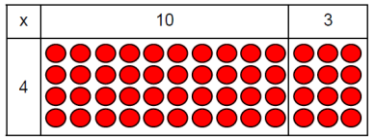
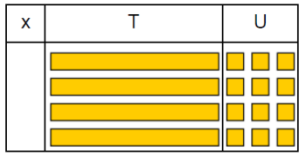
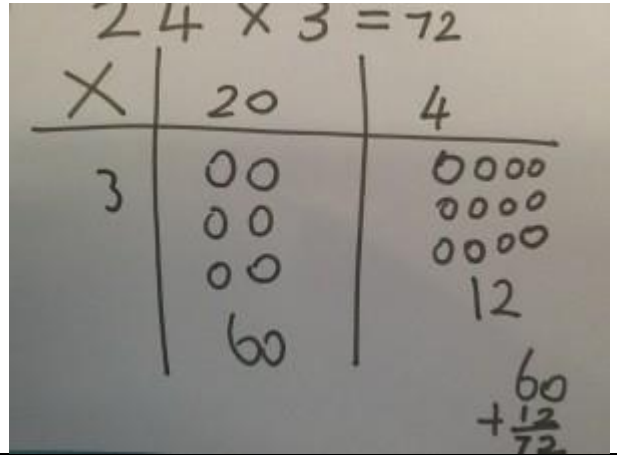
	from 4 easily? I need to exchange one of my tens for ten ones.  Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction	Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	partitioning the number into clear place value columns. Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 ~~6~~ ~~3~~ . **0** - 2 6 . 5 2 3 6 . 5

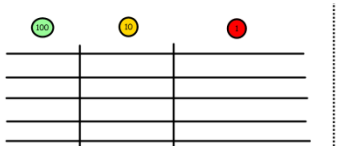
	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

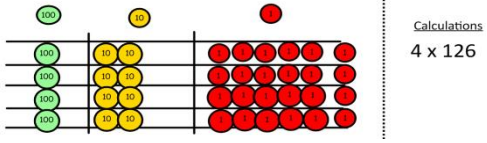
Multiplication

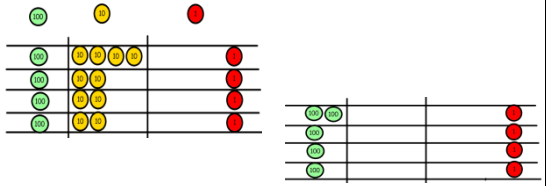
Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number. 	 Partition a number and then double each part before recombining it back together.

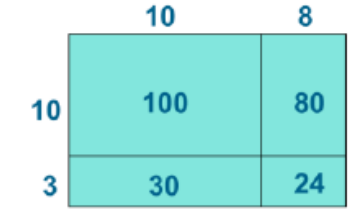
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 

Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.    Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Grid Method	Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | --- | --- | --- | | 7 | 210 | 35 | 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

<table>
<tr>
<td></td>
<td>


Fill each row with 126.


Add up each column, starting with the ones making any exchanges needed.


Then you have your answer.
</td>
<td></td>
<td>


<table>
<tr><th>X</th><th>1000</th><th>300</th><th>40</th><th>2</th></tr>
<tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr>
<tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr>
</table>
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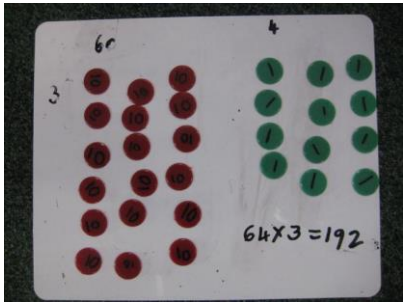
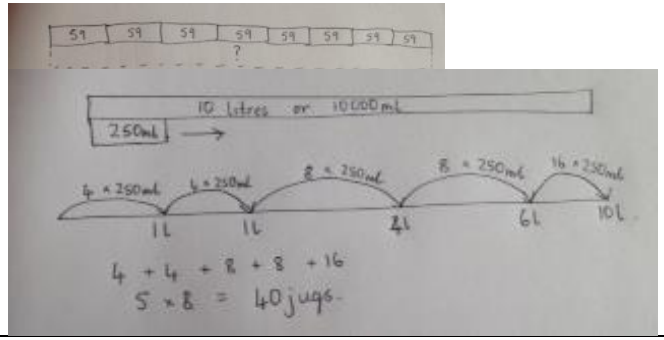
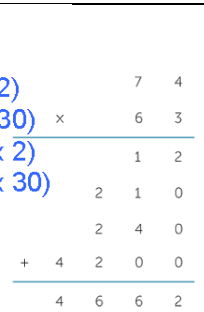
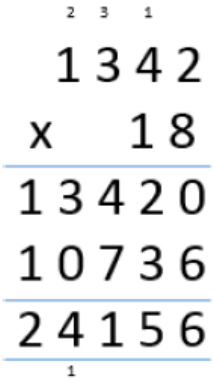
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication.


</td>
<td>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.


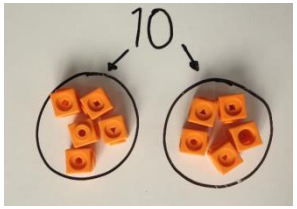
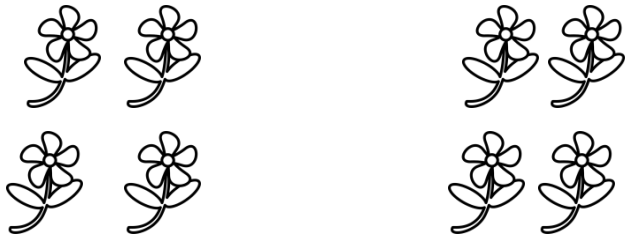
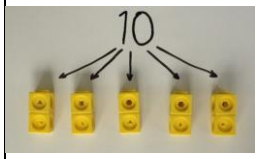
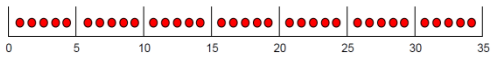
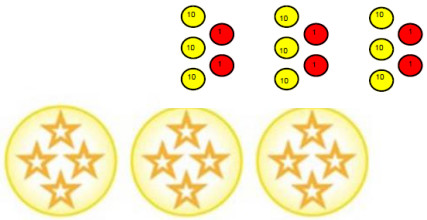
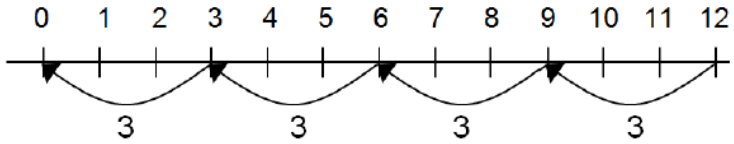
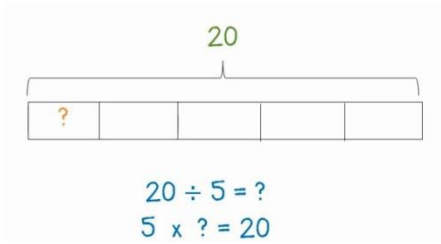
</td>
<td>Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

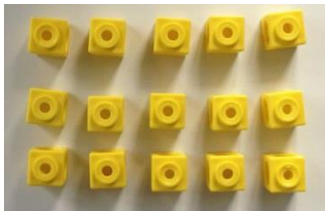
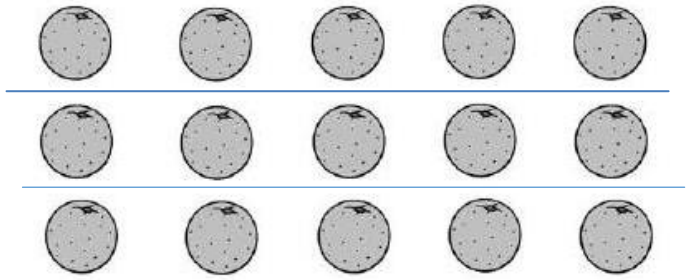
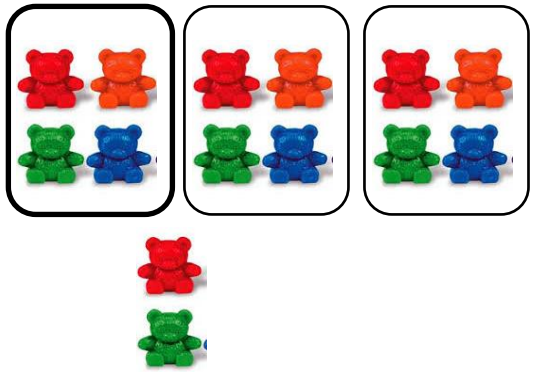
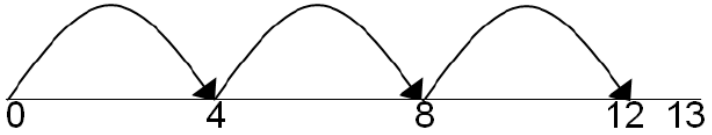
If it helps, children can write out what they are solving next to their answer.</td>
</tr>
</table>

	It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.		32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768  This moves to the more compact method. 

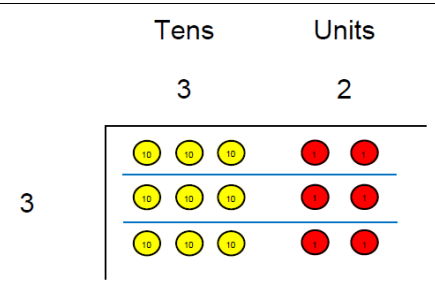
Division

Objective and Strategies	Concrete	Pictorial	Abstract

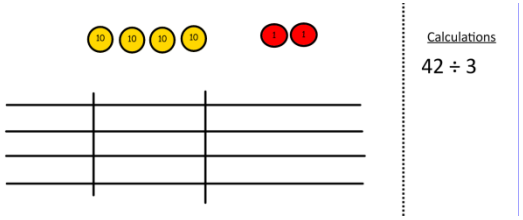
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   $96 \div 3 = 32$ 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 

Short division

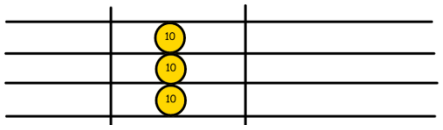


Use place value counters to divide using the bus stop method alongside

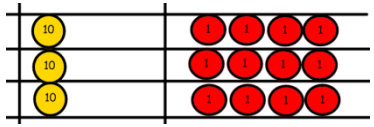


42 ÷ 3=

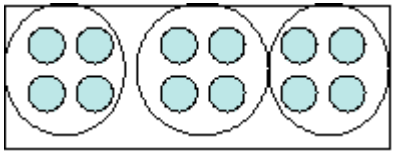
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.

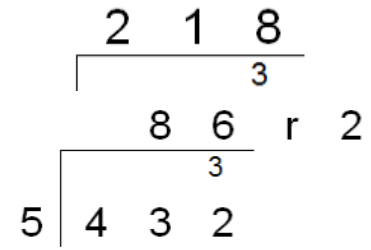


Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



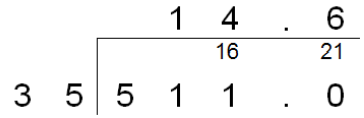
Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.



Move onto divisions with a remainder.

Finally move into decimal places to divide the total accurately.



	We look how much in 1 group so the answer is 14.		